Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Topology

Subject Code:5SC01TOP1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date:03/12/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: Lower limit topology. **1**

b. Define: Homeomorphism. **1**

c. Define: Projection map. **1**

d. Give an example which shows that $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$. **2**

e. Let X be a topological space. Let $A, B \subset X$. If $A \subset B$ then prove that $A' \subset B'$. **2**

Q-2 **Attempt all questions** **(14)**

a. Let $X = R$ and $\tau = \left\{ \begin{matrix} U \mid \text{for } x \in U \text{ there exists } \epsilon > 0 \text{ such that} \\ (x - \epsilon, x + \epsilon) \subset U \end{matrix} \right\}$ then prove that τ is topology on R. **5**

b. Let A be a subset of topological space X and A' be the set of all limit points of A. Then prove that $\bar{A} = A \cup A'$. **5**

c. Prove that projection maps are continuous. **4**

OR

Q-2 **Attempt all questions** **(14)**

a. Let X be any topological space and $\mathcal{B}$ be the basis for the set X. Let $\tau = \{U \mid \text{each } x \in U \ \exists\ B \in \mathcal{B} \ \ni x \in B \subset U\}$ then prove that τ is topology on X. **5**

b. Let (X_1, τ_1) and (X_2, τ_2) be topological spaces. Let $X = X_1 \times X_2$. Define $\beta = \{U_1 \times U_2 \mid U_1 \in \tau_1, U_2 \in \tau_2\}$. Prove that β is a basis for X. **5**

c. Prove that $(A \cap B)^0 = A^0 \cap B^0$. Is it true $(A \cup B)^0 = A^0 \cup B^0$? Justify your answer. **4**

Q-3 **Attempt all questions** **(14)**

a. Let X, Y be topological spaces. $f: X \to Y$. Then prove that following are equivalent 6
(1) f is continuous.
(2) for every subset A of X, then $f(\bar{A}) \subset \overline{f(A)}$
(3) For every closed set $B$of Y, $f^{-1}(B)$ is closed in X.

b. State and prove that Pasting lemma. 4

c. State and prove sequence lemma. 4

OR

Q-3 **Attempt all questions** **(14)**

a. Every compact subset of T_2 space is closed. 5

b. Every closed subset of compact space is compact. 5

c. Prove that continuous image of compact space is compact. 4

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Give an example of topological space which is not compact. 1

b. Define: Hausdorff Space. 1

c. Define: Disconnected space. 1

d. Is disconnectedness property ahereditary property? 1

e. Define: Locally compact space. 1

f. Is it true, "Product of Lindelof space is Lindelof "? 1

g. Define: Separable space. 1

Q-5 **Attempt all questions** **(14)**

a. Let X be a topological space. Then prove that X is disconnected space if and only if there exists a non-empty proper subset of X which is both open and closed. 5

b. Define: Path connected. Prove that every path connected space is connected. 5

c. Prove that continuous image of connected space is connected. 4

OR

Q-5 **Attempt all Questions** **(14)**

a. State and prove Heine – Borel theorem. 7

b. If X is second countable topological space then prove that X is separable. 4

c. Prove that every compact space is locally compact. 3

Q-6 **Attempt all questions** **(14)**

a. Define: Second countability axiom. Prove that subspace of second countable space is second countable. Is it true, "Every first countable space is second countable"? Justify your answer. 6

b. Define: Compact space. State and prove extreme value theorem. 6

c. State Urysohn's lemma. **2**

OR

Q-6 **Attempt all Questions** **(14)**

a. Prove that product of T_2 space is T_2. Is it true for normal space? Justify your answer. **6**

b. Define: Completely regular space. Let X be completely regular space and $A \subset X$ then prove that A is completely regular space. **6**

c. State: Tychonoff theorem. **2**